Voyager & Ozobot: Teaming Up to Study Kepler's Law of Equal Areas

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In the early 1600's Johannes Kepler discovered three empirical laws regarding the motion of planets "around" the sun. The second law, known as the *Law of Equal Areas*, states that a line connecting any planet and the sun sweeps out equal areas in equal times. Although there are a number of Web-based screen animations illustrating Kepler's Law of Equal Areas, there are virtually no widespread physical demonstrations using actual hardware—at least not until Ozobot made the scene! Now with Voyager and Ozobot working *together as a team*, the motion can be visualized *and studied quantitatively*. There is no better way to experience Kepler's Law of Areas first-hand than by observing an actual physical object move according to this law. See Figure 1 for a snapshot of the Voyager/Ozobot team as it travels around the ellipse collecting angular velocity data. A small piece of modeling clay keeps Voyager from falling off of Ozobot. A video captured from the PocketLab app entitled *Kepler2ndLaw.mp4* accompanies this lesson—50 data points per second.

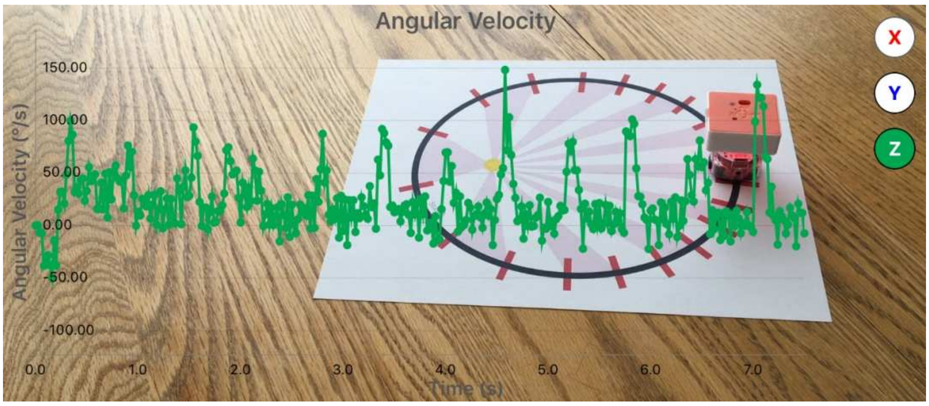


Figure 1

A full page "Ozomap" that can be printed for actual use with Ozobot and Voyager appears on the last page of this document. The solid black oval is the ellipse representing the orbit of a planet, asteroid, comet, or meteoroid, in its orbit with the sun at one focus of the ellipse, as per Kepler's first law, the *Law of Orbits*. The red and while "triangles" (they actually are curved on one side, so calling them triangles is an approximation) are all of roughly equal area. The red hash marks along the ellipse separate roughly equal time intervals as Ozobot travels about the focus where the sun is located. In order to carve out equal areas when nearest (at *perihelion*) to the sun, Ozobot must travel faster than when it is furthest (at *aphelion*) from the sun. For the ellipse on the Ozomap, Ozobot's speed at perihelion is approximately three times its speed at aphelion. The ability to adjust Ozobot's speed programmatically via OzoBlockly makes this demonstration possible. Ozobot will adjust its speed each time it crosses one of the red hash marks on the ellipse. The result is a very realistic physical demonstration of Kepler's Law of Areas.

Figure 2 shows the OzoBlockly program that is loaded into Ozobot. A subroutine called "do something" uses its input parameter *x* to set the speed of Ozobot as it follows the ellipse. Ozobot follows the ellipse to the next intersection and picks the direction to go straight, staying on the ellipse. The main program sets the color of the robot's top light to red and then gives the student two seconds to place Ozobot at the start location on the ellipse. Following that is simply a loop with a sequence of calls to the *do something* subroutine, with Ozobot's speed adjusted with each call. The calls to the subroutine are in a repeat forever loop. Therefore, Ozobot will traverse the ellipse over and over again until the student turns Ozobot off or until the battery has been depleted.

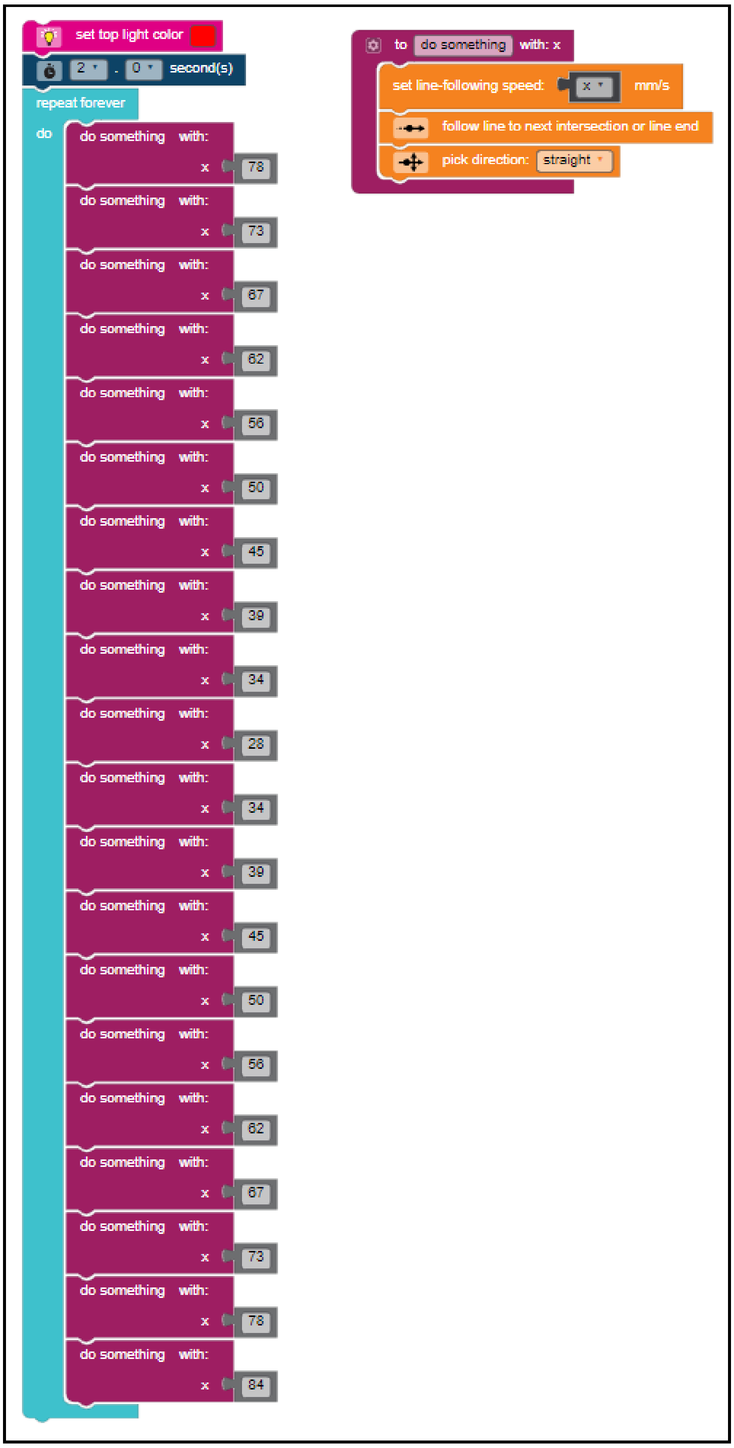


Figure 2

The average angular velocity of the earth around the sun is 360°/year or a little less than one degree per day. Because the earth's orbit is ever so slightly elliptical, the actual velocity, however, varies a little throughout the year. It travels the fastest during the Northern Hemisphere's winter when it is at perihelion and the slowest during the summer when it is at aphelion. (That's right—the earth is closer to the sun in the winter than the summer, but it is not distance from the run that determines the seasons. It is the tilt of the earth's axis relative to the plane of its orbit.)

Figure 2 shows an ellipse with definitions of some major characteristics of the ellipse. F and F' are the foci. VV' is the major axis. UU' is the minor axis. a and b are the length of semi-major, and semi-minor axes, respectively. The eccentricity of an ellipse is a measure how close to being circular it is. The eccentricity of a circle is 0.

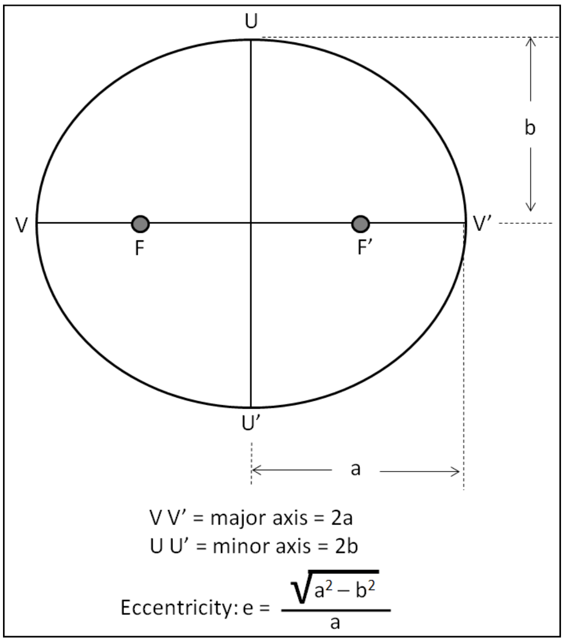


Figure 3

Students could take measurements of the semi-major and semi-minor axes of Ozobot's ellipse in order to calculate it eccentricity. *[The eccentricity will be about 0.5.]* They could then consult the *Wikipedia article "List of periodic comets" and find a number of comets in that list that are close to the eccentricity of Ozobot's ellipse.* One possible comet has a designation *P/2007 T6*, named *Catalina (CSS)*. This comet has an eccentricity of 0.50286, a period of 9.51 years, and a perihelion of 2.2322 AU (Astronomical Units). Its last perihelion was on February 18, 2017.

Figure 4 shows a graph of angular velocity vs. time for the Ozobot/Voyager team. This graph was created in Excel from a csv file produced by the PocketLab app. Although there is a lot of variation in the blue line due to vibration while moving, it clearly shows that the angular velocity varies with time. The *Add Trendline* feature of Excel, using the Trend/Regression Type of *Moving Average* (averaged over 50 data points), shows that the angular velocity at perihelion is about twice that at aphelion. It appears that high eccentricity results in more significant variations in angular velocity over the period of a comet as it travels around the sun.

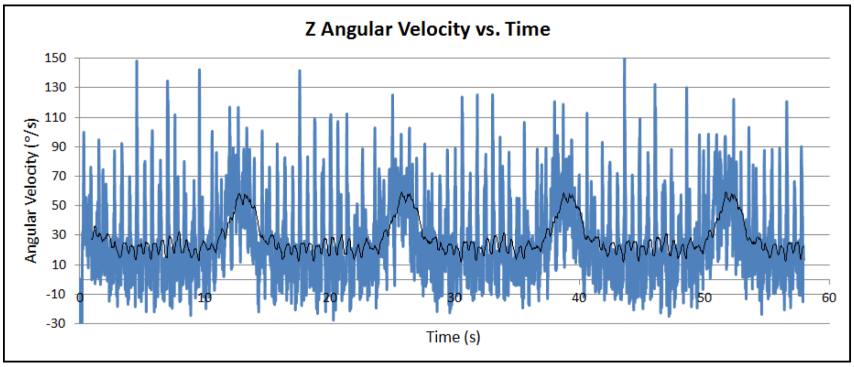
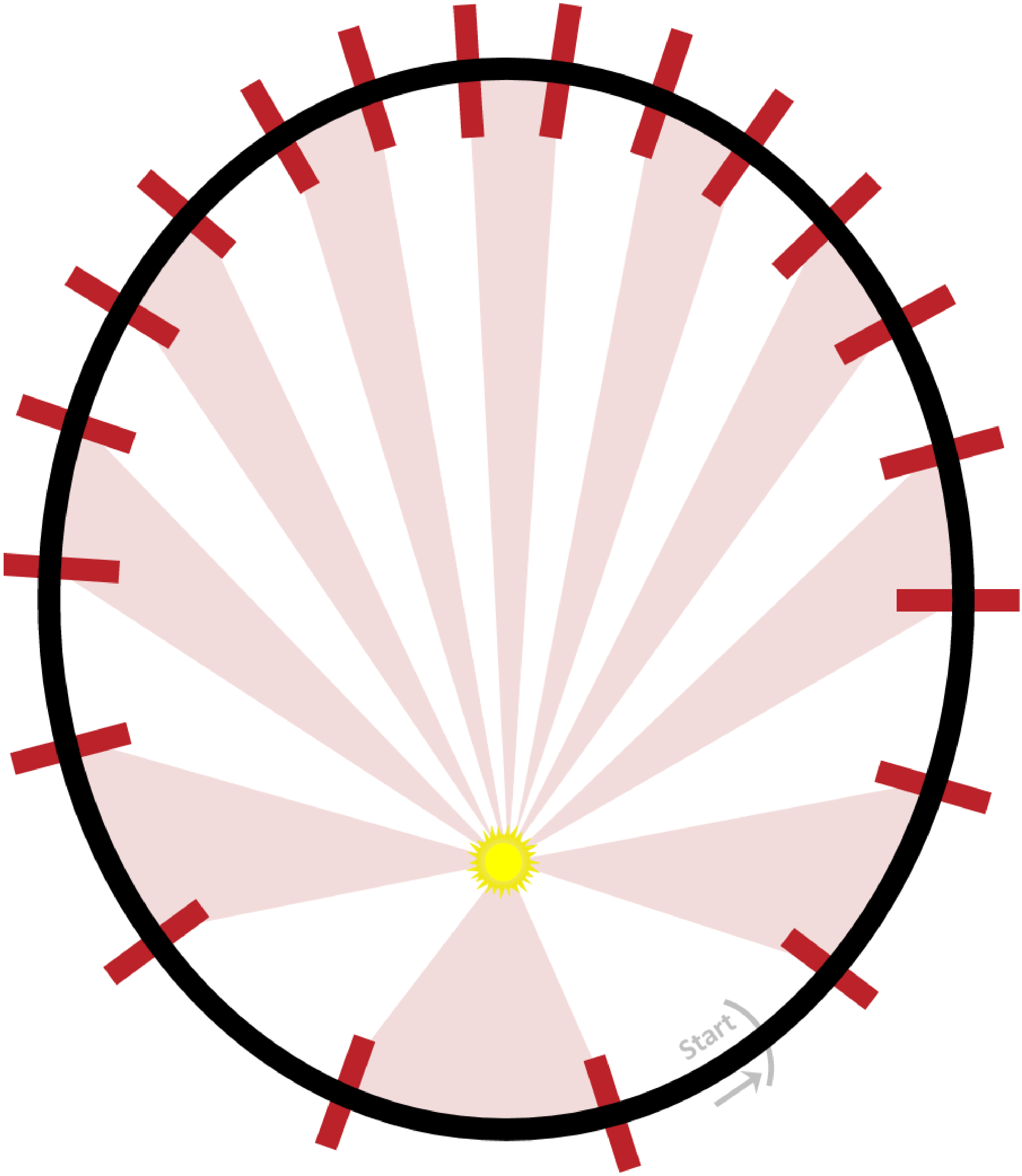


Figure 4

It your students actually do this investigation using Voyager and Ozobot, they should be able to explain how the angular velocity vs. time graph provides good evidence that the times between intersections on the ellipse are reasonably close to being equal—an important part of the Law of Equal Areas. *[Hint: What does Ozobot do each time that it encounters an intersection? How does this show up in the angular velocity vs. time graph?]*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