PocketLab Voyager: A Study of Coupled Pendulums

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Introduction

It is generally believed that Galileo was the first person to perform scientific studies of the pendulum. In the early 1600's he found that the period is proportional to the length of the pendulum. He also observed that the period is not affected by the mass of its bob and is roughly independent of the angle of its swing. In the mid 1600's Huygens invented the pendulum clock, which remained a major time-keeping instrument until quartz watches and atomic clocks made their appearances. The most famous pendulum is probably the Foucault pendulum, which can be used to demonstrate that the Earth rotates.

This lesson deals with what are commonly referred to as ***coupled pendulums***, in which energy is transferred back-and-forth between the pendulums via the coupling. Observations of two pendulum clocks on a mantle in the mid 1800's by Huygens provide an early example of coupled pendulums. Thermal vibrations at the molecular level also represent coupled oscillations. Pendulums coupled by springs are commonly studied in college physics classes during studies of simple harmonic motion. However, our lesson makes use of string-coupled pendulums, as they are easier and less expensive to construct, as shown in Figure 1.

Figure 1

The pendulum bobs are wood blocks that have been taped to balsa wood sticks (~0.5 meters in length) using removable mounting tape. Voyager has been taped to one of the bobs. The balsa sticks have been taped to a string with ordinary masking tape. Finally, each end of the string is taped to a support, in this case an entryway. The balsa sticks were selected as they help to keep each pendulum swinging in a plane without precession.

The Normal Modes of a String-Coupled Pendulum System

There are two modes of vibration of a coupled pendulum system that physicists refer to as ***normal modes***. These are shown and explained in an accompanying video *Modes.m4v* for our string-coupled pendulums. These normal modes are important as they yield the steady-state solution for a coupled pendulum when superimposed. The two modes are obtained as follows:

- ***Symmetric Mode.*** The pendulums are in phase with one another, moving side-by-side. In this mode, the coupling string transmits no force. Students can take measurements to show that for the symmetric mode, the period of the coupled pendulums is the same as each pendulum if they were not connected.
- ***Antisymmetric Mode***. The pendulums move in opposition, 180º out-of-phase with one another. In this mode, the coupling string exerts a restoring force that increases the pendulum's speed slightly. This, in turn, results in a decrease in the period, or equivalently, an increase in the frequency of vibration.

When these two modes are superimposed by pulling one pendulum back while the other is kept stationary (as shown in the video), the motion of the normal modes gradually gets out-of-step with one another. Energy is transferred back-and-forth between the pendulums, resulting in beats. *In this lesson, students show that the difference in the frequencies of the symmetric and antisymmetric mode is equal to the beat frequency*.

Experimental Results

Figure 2 shows a snapshot of the beats produced when the two modes are superimposed. This image was obtained from the PocketLab app using the angular velocity sensor with a 20 points per second data collection rate. With Voyager mounted as shown in the figure, the Y angular momentum is of interest, since Voyager is moving back-and-forth in the X-Z plane. See the accompanying movie *CoupledPendulums.mp4* for all the action!

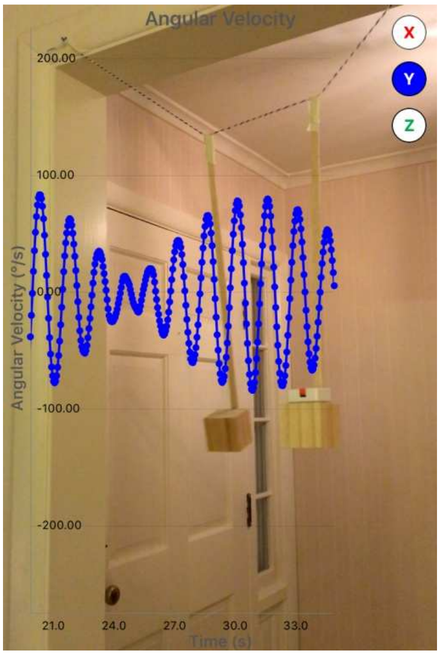

Figure 2

Figure 3 shows charts that were constructed using Excel from csv files obtained from the PocketLab app and Voyager angular velocity sensor. All three show a gradual decrease in amplitude, as would be expected, due to frictional forces in the system. The difference in the frequencies of the symmetric mode and antisymmetric mode is 0.697 Hz – 0.649 Hz = 0.048 Hz. This is within about 10% of the beat frequency (0.053 beats/s) of the coupled pendulums.

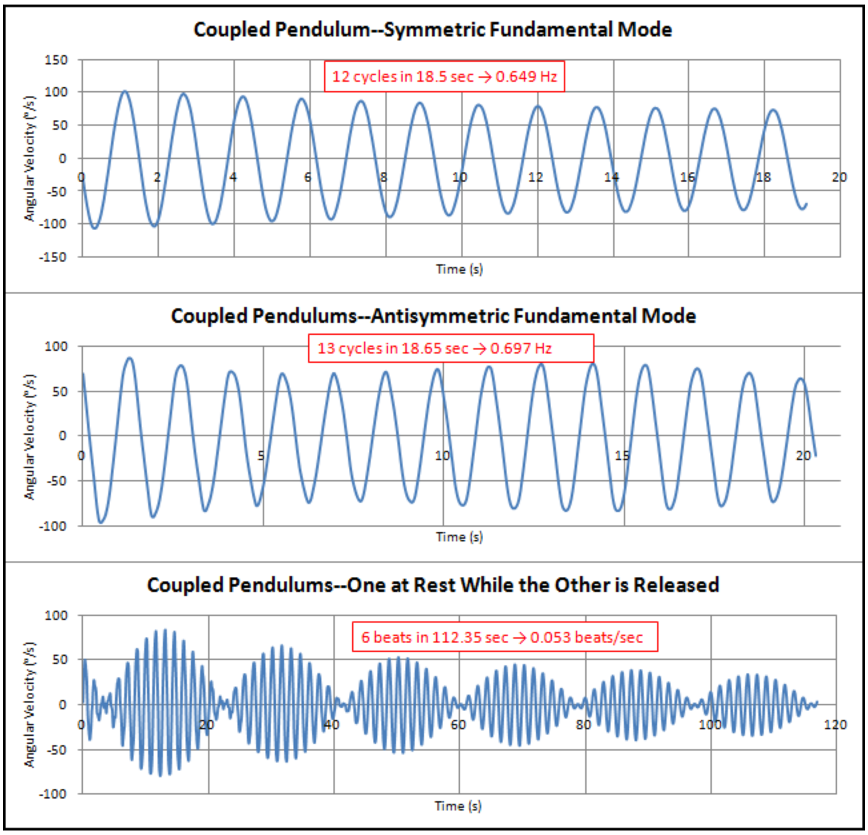

Figure 3

Further Investigations

Students can experiment with a variety of independent variables to see how they affect the interdependency of the coupled pendulums:

- Length of the pendulums
- Tension in the string
- Masses of the bobs

Remark on the Mathematical Theory of Coupled Pendulums

A detailed study of the mathematical theory of coupled pendulums may require mathematics learned in upper-level college courses. However, if the purpose is to *investigate the behavior* of coupled pendulums with an emphasis on the physical meaning of the normal modes and their interaction to produce beats, then this study is certainly within the purview of high school AP physics.