PocketLab Voyager: Moment of Inertia and Conservation of Angular Momentum

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As shown in Figure 1, conservation of angular momentum can be investigated using a Lazy Susan (LS), PocketLab, and a compact weight. Voyager is mounted to the LS using double stick tape or a small piece of modeling clay. With the angular velocity sensor selected at a data rate of 50 points per second, the LS is given a spin. The LS gradually slows down, primarily due to friction in its bearings. When it has slowed down enough so that the compact weight will not fly off when dropped just above the edge of the LS, the weight is released. The resultant sudden decrease in angular velocity is recorded by Voyager. An accompanying video shows all of this action. Taking into account the moment of inertia of the LS, and approximating the compact weight as a point object at a known distance from the center of the LS, the law of conservation of angular momentum can be verified to within a few percent.

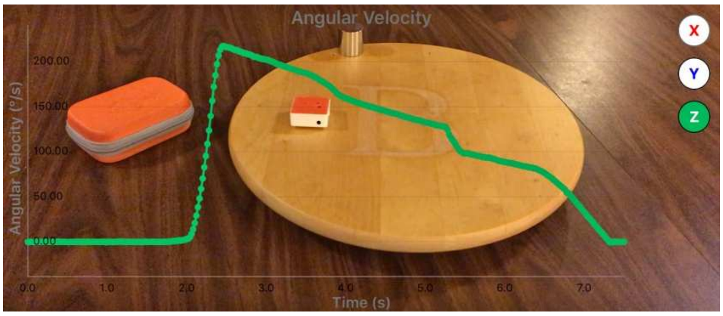

Figure 1

Figure 2 shows an Excel chart of angular velocity vs. time that was created from the PocketLab app *csv* file. The chart has been marked with comments explaining what is happening.

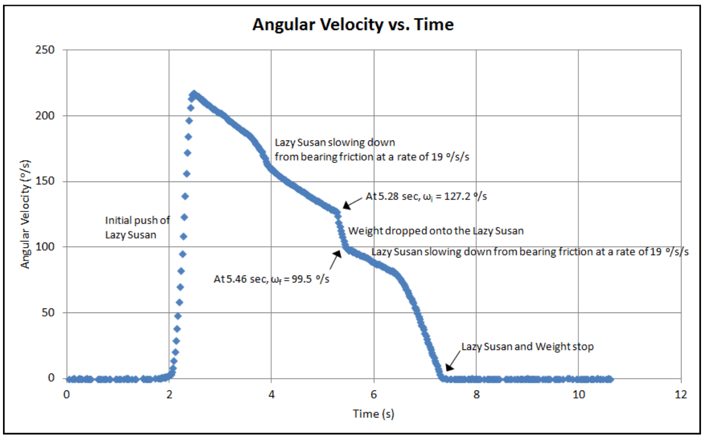

Figure 2

The graph of Figure 2 shows how the initial angular velocity (127.2 °/s) and final angular velocity (99.5 °/s) were obtained from direct readings on the graph. With these two readings in mind, Figure 3 provides the equations used along with a sample calculation of the initial and final angular momentums. We see that the initial and final angular momentums are within 4.44% of one another.

Initial angular momentum = $L_i = I_{LS}\omega_i$,
where I_{LS} = moment of inertial of the Lazy Susan about its axis,
and ω_i = angular velocity of Lazy Susan just before releasing the weight

Final angular momentum = $L_f = I_{LS}\omega_f + I_W\omega_f$,
where ω_f is the final angular velocity of the Lazy Susan and weight,
I_{LS} is the moment of inertial of the Lazy Susan about its axis,
and I_W is the moment of inertial of the weight considered as a point object

$I_{LS} = \frac{1}{2}MR^2$, where M is the mass and R is the radius of the Lazy Susan
$I_W = mr^2$, where m is the mass of the weight, and r is the distance from the center of the Lazy Susan to the center of the weight.

Sample data:
M = 1.405 kg
m = 0.200 kg
R = 0.1905 m
r = 0.168 m
ω_i = 127.2 °/s * π/180 rad/° = 2.22 rad/s
ω_f = 99.5 °/s * π/180 rad/° = 1.74 rad/s

L_i = 0.0565 kg-m²/s
L_f = 0.0541 kg-m²/s

% difference
= |0.0541 – 0.0565|/0.0565
= 4.4%

Figure 3

An Improvement in Our Results

It was noted in Figure 2 that the angular velocity was slowing down at a rate of about 19° /s/s just before and just after the weight was dropped. It is reasonable to assume that it was also slowing down at this rate *during the impact* of the drop. This means that the final angular velocity would have been higher than the 99.5 °/s shown in Figure 3. Based upon data provided in Figure 2, the time for the impact was 5.46 s – 5.28 s = 0.18 s. Multiplying this time by 19° /s/s, we get a 3.42 °/s loss in angular velocity that is not due to the impact of the weight. Therefore, the final angular velocity would be 99.5 °/s plus 3.42 °/s = 102.9 °/s. This results in a %-difference of angular momentum before and after of only 1.2%!