PocketLab Voyager/LEGO®: A Study of the Atwood Machine

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Introduction

In the late 1700's, the Englishman George Atwood devised what is now known as the ***Atwood machine***. Students since then have studied this machine to verify Newton's Second Law of Motion. In this machine, two hanging masses are tied to the end of a string that loops around a pulley. The larger mass then moves downward with a constant acceleration, while the smaller mass accelerates upward. The magnitude of this acceleration is a quantity of great interest as it relates to the values of the two masses.

With accelerations typically between 0 and 1 m/s^2, and Voyager accelerometer resolution on the order of 0.1 m/s^2, acceleration data is quite noisy. A much better method is to collect position data for one of the two masses using Voyager's IR range finder. A quadratic fit $x = At^2 + Bt + C$ can then be applied to the resulting position vs. time graph. Since velocity $v = dx/dt$, we have $v = 2At + B$. Since $a = d^2x/dt^2$, we have $a = 2A$. So, twice the value of A in our quadratic fit gives the acceleration!

In this lesson, an Atwood machine, as shown in Figure 1, is constructed using parts from LEGO®'s Simple & Powered Machines Set. Voyager is mounted (using VELCRO®) to the bottom of the larger mass with its IR sensor facing a large piece of white foam board on the floor. The pulley holder is mounted to the table using picture hanging strips. Gears from the LEGO® set serve as the two masses. Voyager is mounted on the larger of the two masses, and is itself part of the larger mass.

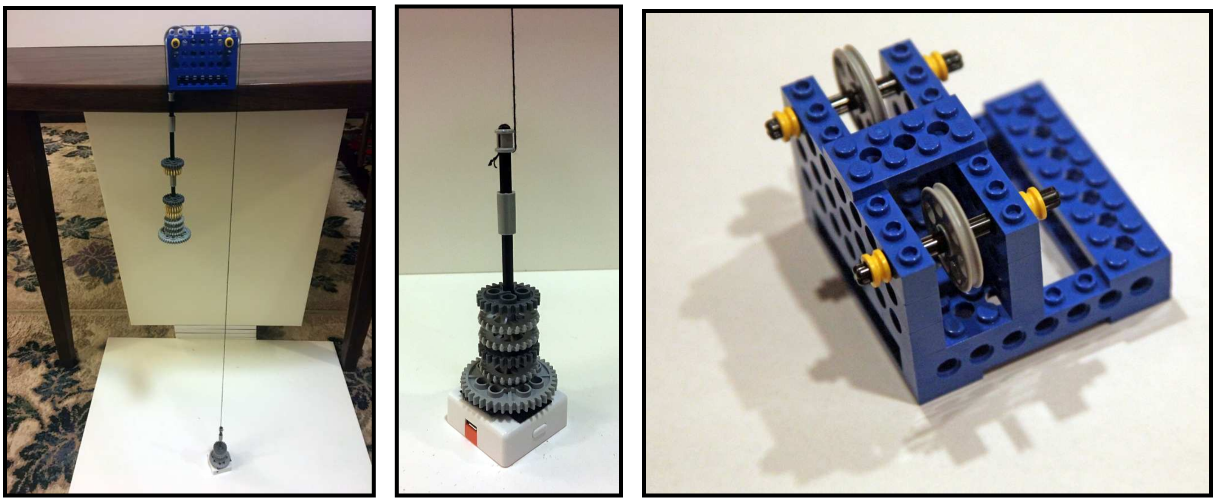

Figure 1

Atwood Machine Dynamics

Figure 2 shows the free body diagrams and equations of motion for an ideal frictionless Atwood machine. We assume that the pulley is frictionless and massless, and that the string is massless. Further we assume that $M > m$ so that m will accelerate upward. We will take upward acceleration as positive. Therefore, if the acceleration of m is a, then the acceleration of M is $-a$. The resulting equations for Newton's Second Law of Motion, when combined using a little bit of algebra, yield the important "blue-star" equation.

This "blue-star" equation provides us with the beauty of the Atwood machine. The acceleration of gravity is constant. The equation therefore tells us that the acceleration is proportional to the difference in the masses ($M - m$) *if we keep the sum of the masses $M + m$ constant*. How do we accomplish this? We do this **by transferring mass from mass *m* to mass *M***. With our LEGO® setup, transferring mass simply means moving LEGO® gears from m to M. Each time we transfer some mass, we measure the value of the two masses and use Voyager's range finder to determine the acceleration. It is our hope that a graph of acceleration vs. mass difference will be a straight line, providing good evidence that *acceleration* is proportional to the *difference in masses*.

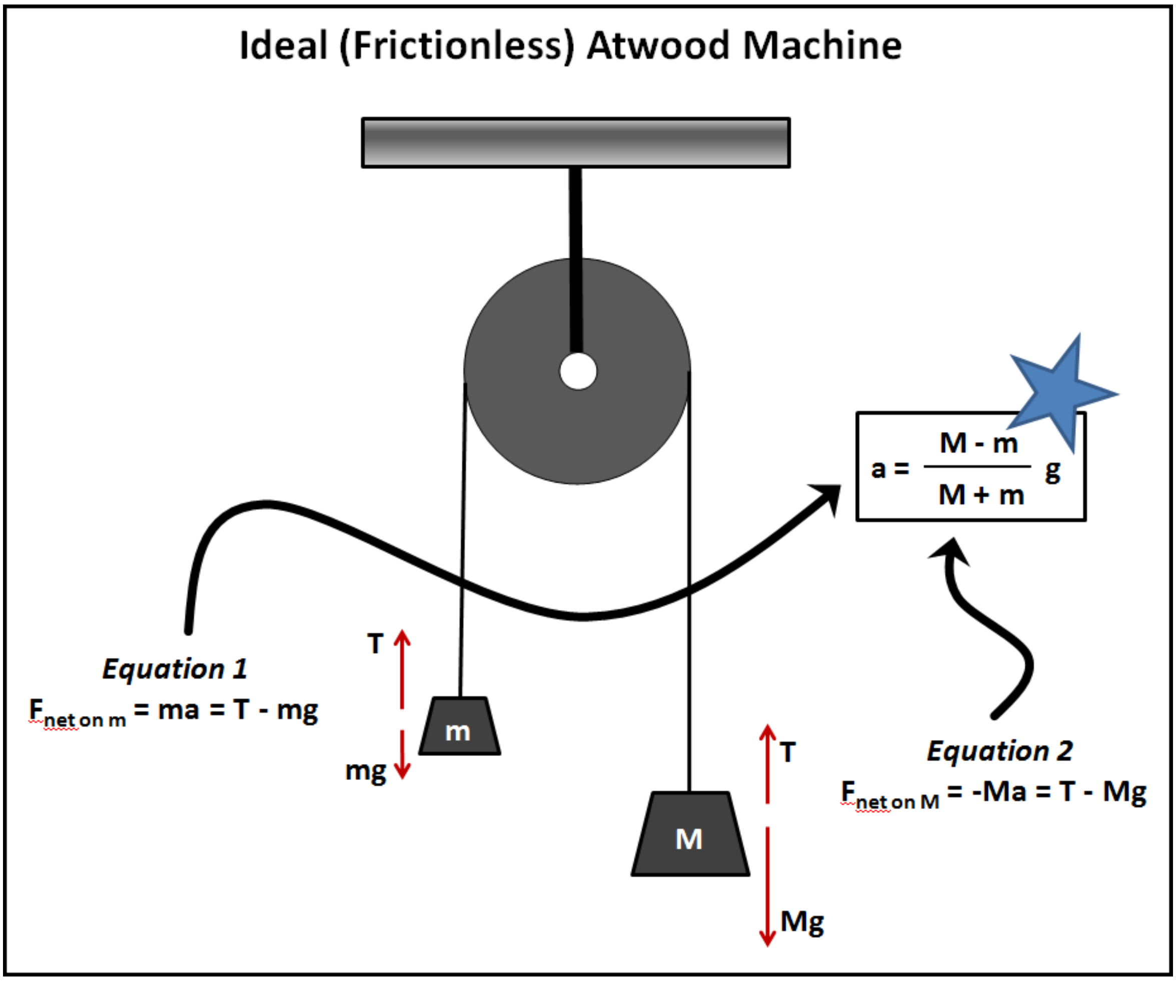

Figure 2

Data Analysis

Figure 3 contains two graphs that were constructed in Excel from Voyager range finder data obtained from the csv file created by the PocketLab app. The graphs correspond to the data run for which the hanging mass *m* = 19.33 g and *M* = 26.87 g. The top graph shows that the masses were released at about 0.76 seconds, accelerated, and finally stopped at about 1.66 seconds. The region shown in red was enlarged in the bottom graph for the purpose of performing a quadratic curve fit on the region of acceleration. Based upon our discussion in the introduction, we find that the acceleration of mass M is about 2 x -0.5108 = -1.02 m/s^2. Alternatively, the acceleration of mass m is about 1.02 m/s*2*.

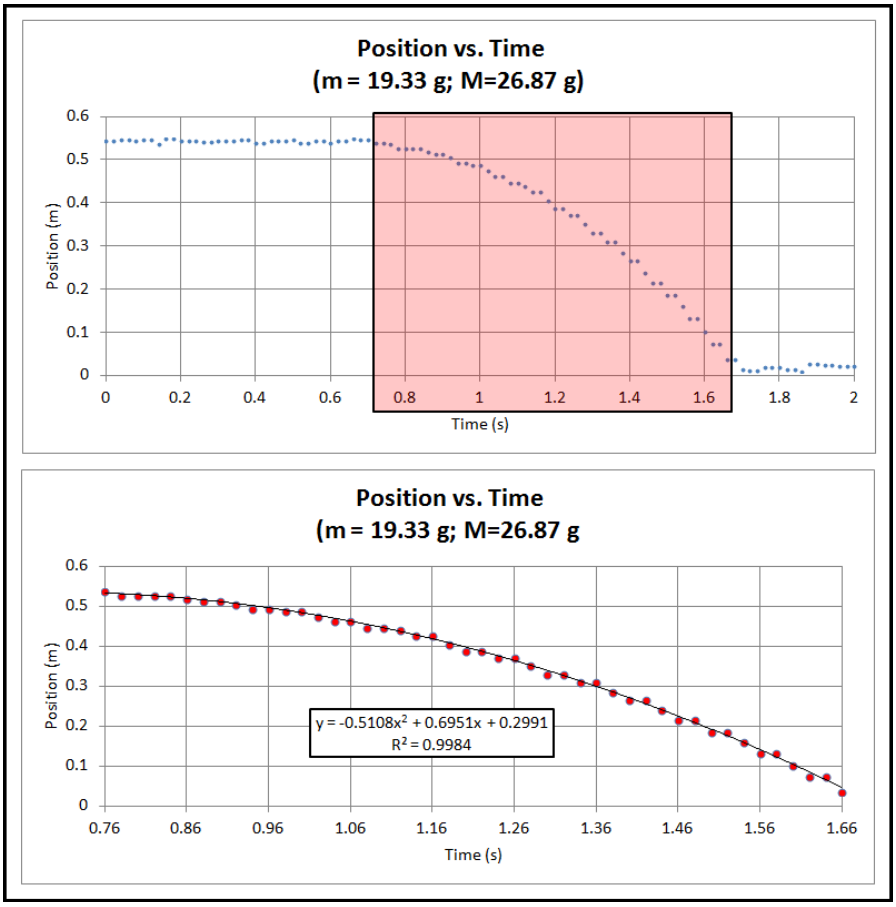

Figure 3

Figure 4 shows an Excel chart that summarizes the *(M -m, acceleration)* data pairs that were obtained by the procedure outlined in the example of Figure 3. A linear fit to the data, with an R^2 value of 0.997, provides strong evidence that the acceleration is proportional to the mass difference *M - m*. The acceleration increases by about 0.2 m/s^2 per gram of additional mass difference. We note that the x-intercept of the straight line of best fit occurs at about 3 g. Although we have attempted to minimize friction by using a low friction pulley, friction is never-the-less still present. A minimum of 3 g for the mass difference is required just to overcome frictional forces involved with our Atwood machine.

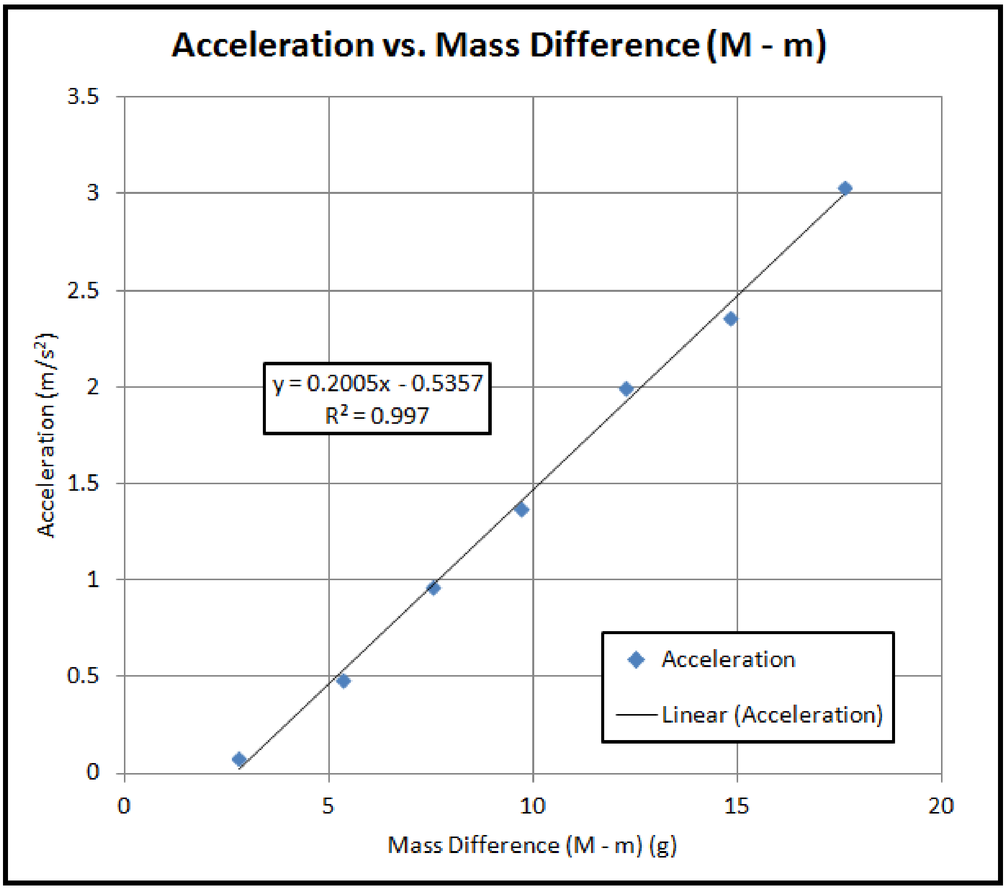
Acceleration vs. Mass Difference (M - m)
3.5
3
2.5
2
1.5
1
0.5
0
0
5
10
15
20
Acceleration (m/s²)
Mass Difference (M - m) (g)
y = 0.2005x - 0.5357
R² = 0.997
Acceleration
Linear (Acceleration)

Figure 4