Physics from a Croquet Mallet and Ball

Teacher's Guide

If your students need a hint on how to proceed in this lesson, it could be noted that the list of requested items have been listed in a natural order for calculation.

Figure 1 below shows the values of required masses and length of the mallet pendulum used by the author throughout all calculations. Unlike a typical pendulum bob suspended by a light string, the mass of the mallet's handle is not negligible. However, it was less than the mallet at the bottom of the handle. Therefore, the decision as to what to use for the mass of the mallet can be challenging. The author was able to unscrew the handle and measured the mass of the mallet (plus Voyager) at the bottom of the handle. In effect, the author assumed negligible mass for the handle. In reality, some mass value between that of the mallet at the bottom of the handle and the mass of the entire handle plus mallet might be better.

The length of the pendulum *h* is 0.385 m, the mass of the ball *m* is 0.213 kg, and the mass of the pendulum bob (including Voyager) *M* is 0.223 kg. We note that the mass of the ball is slightly less than that of the pendulum.

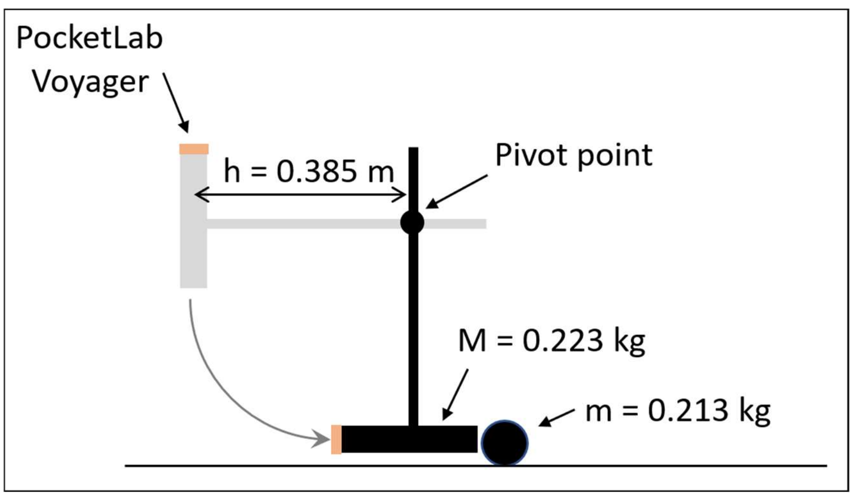

Figure 1

The Speed of the Mallet Just Before Striking the Ball

Conservation of energy provides a convenient method for determining the speed of the mallet just before striking the ball. The *gravitational potential energy* when the pendulum is held horizontally at the top is converted into *kinetic energy* when the pendulum is vertical just before striking the ball:

$$Mgh = ½Mv^2 \rightarrow v = SQRT(2gh),$$

where g is the acceleration of gravity, and v is the velocity of the mallet just prior to impact with the ball. Therefore, v = SQRT(2 * 9.8 m/s^2 * 0.385 m) = 2.75 m/s. Air resistance and friction at the pivot point are assumed to be negligible.

The Maximum Acceleration of the Mallet During Impact with the Ball

The maximum acceleration of the mallet during impact with the ball can be obtained from Voyager's accelerometer data. The author set the data rate to the maximum allowable value of 50 points/sec. Dependent on the exact instant of time during the impact at which a data point is captured, there will be some variance in the acceleration value from one run to the next. Therefore, it is desirable to drop the pendulum several times and find the average of the maximum acceleration values for all of the strikes of the mallet on the ball. Figure 2 shows an Excel graph of acceleration vs. time for ten drops of the pendulum. The orientation of Voyager attached to the mallet makes the z-acceleration of interest here. Note that as the pendulum is raised to horizontal, the z-acceleration approaches 10 m/s^2. Students should be required to explain why it is shown as 10 m/s^2 when the pendulum is being held still. At the bottom of the swing, just before impact with the ball, the z-acceleration is 0, as we would expect at the bottom of the swing. The ten sudden downward spikes represent the maximum acceleration recorded during each impact. The highest and lowest readings were discarded by the author, and the remaining eight acceleration values (shown in red) were averaged. This average value for the maximum acceleration during impact is given by a = 24.25 m/s^2.

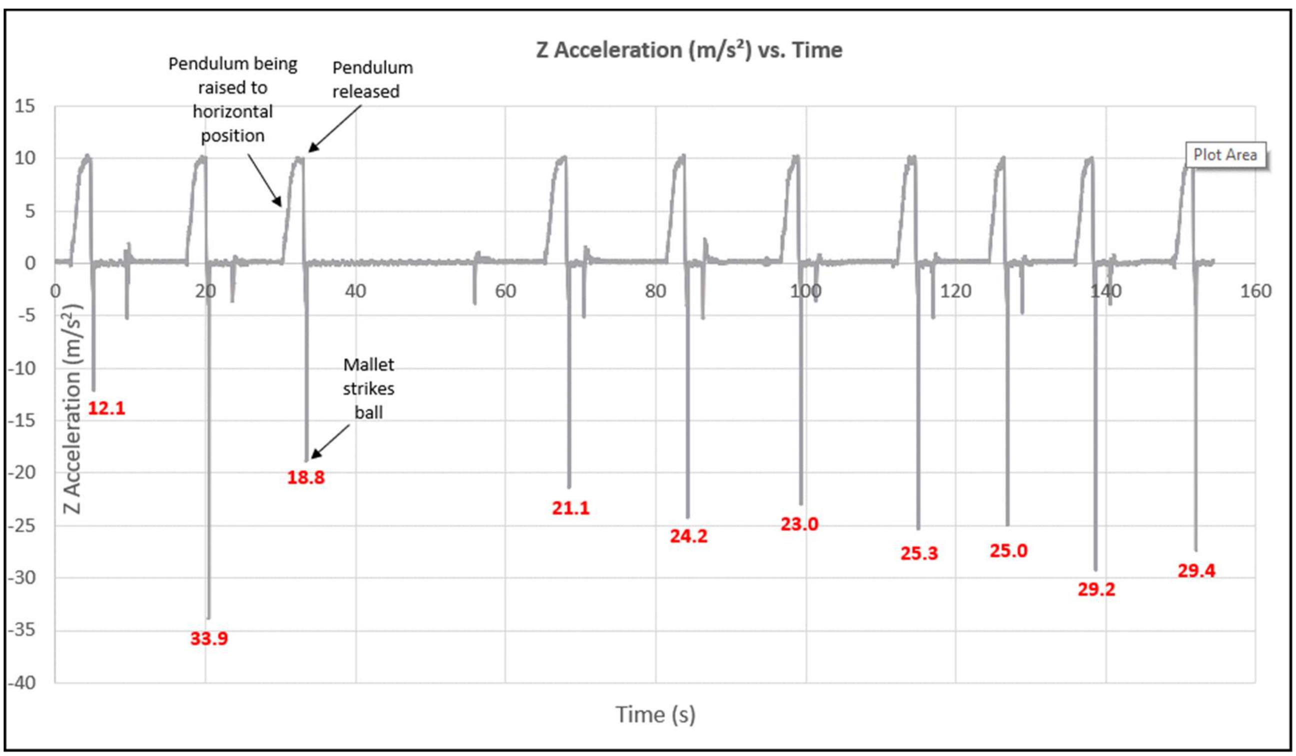

Figure 2

The Maximum Force of the Mallet During Impact with the Ball

We make use of *Newton's 2nd Law of Motion* to determine the maximum force of the mallet during impact with the ball:

$$F = ma = (0.223 \text{ kg}) * (24.25 \text{ m/s}^2) = 5.41 \text{ N}.$$

The Maximum Force of the Ball During Impact with the Mallet

Newton's 3rd Law of Motion tells us that the force of the ball is equal but opposite in direction to that of the mallet. Therefore, the magnitude of this force is also 5.41 N.

The Maximum Acceleration of the Ball During Impact with the Mallet

We again use *Newton's 2nd Law of Motion* to determine the maximum acceleration of the ball during impact with the mallet:

$$F_{ball} = ma_{ball} \rightarrow a_{ball} = F_{ball}/m = 5.41 \text{ N} / 0.213 \text{ kg} = 25.4 \text{ m/s}^2.$$

The Speed of the Croquet Ball Following Impact with the Mallet

The combined video/data obtained from the PocketLab app is key to determining the speed of the croquet ball following impact with the mallet. A frame by frame analysis of the ball rolling away can be performed using many video editors. Figure 3 on the next page shows snapshots of six consecutive frames of the video. You can see the motion-blurred yellow ball gradually moving off to the right. Its position in each frame can be read along the meter stick with the aid of the 10-cm markers on the tabletop. These positions can then be plotted against time in an Excel graph similar to that shown in Figure 4. The snapshots are taken at 1/30th of a second interval. This is because the PocketLab app takes frames in a video at the default rate of the iPhone camera. The slope of a linear trendline of the position versus time graph tells us the velocity of the ball after impact from the mallet. Figure 4 shows us that this velocity if about 3.29 m/sec. We note that this is somewhat larger than the speed of the mallet just before impact. This seems reasonable as the mass of the ball is somewhat less than the mass of the mallet.

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Figure 3

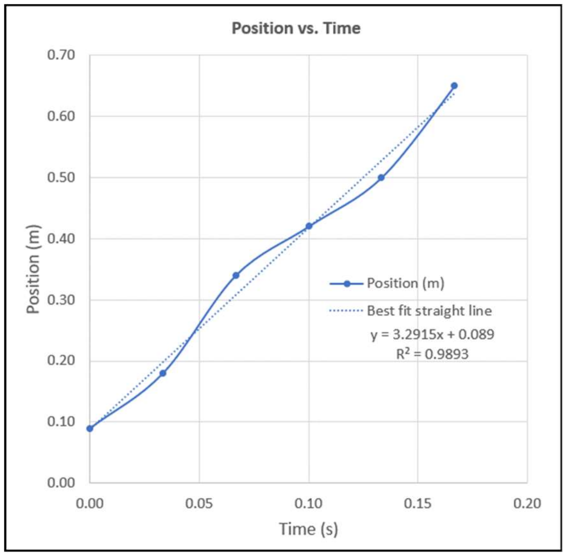

Figure 4

The Amount of Time During the Actual Impact of the Mallet and Croquet Ball

From *Newton's 2nd Law of Motion*, $F = ma$, and the fact that $a = \Delta v / \Delta t$, we can see that:

impulse = change in momentum

or, $F\Delta t = m\Delta v$.

The impulse received by the croquet ball is equal to its change in momentum. Since we know F, m, and Δv, we can readily determine the impact time Δt. Since the ball is initially at rest before the impact, $\Delta v = v_f - v_i$ = 3.29 m/s – 0 m/s = 3.29 m/s. Therefore,

$$\Delta t = (m * \Delta v) / F = (0.213 \text{ kg} * 3.29 \text{ m/s}) / 5.41 \text{ N} = 0.13 \text{ s}.$$

So, the actual impact appears to last approximately 0.1 second. It is worth zooming in on one of the impacts shown in Figure 2 to see if there is agreement with our 0.1 second time. This has been done with the impact between 33 and 34 seconds and is shown in Figure 5. The most intense impact does appear to be on the order of 0.1 second.

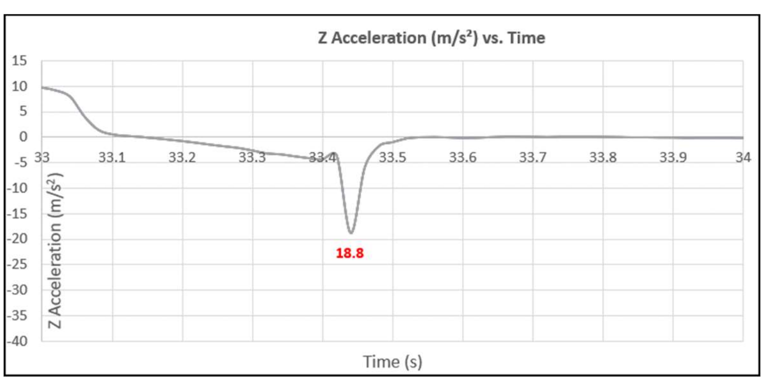

Figure 5