Rotational Motion and Moment of Inertia

Teacher's Guide

Design a PocketLab-based experiment to determine the speed of the center of mass when each of the objects reaches the bottom of the ramp. Which app do you feel would provide for the easiest and quickest analysis—the PocketLab app or the VelocityLab app?

Any experiment design should definitely require several trials for each of the three objects rolling down the ramp. Averages would then be used for analysis purposes. It is probably best to record all data at the highest rate possible—50 points/sec. Figure 1 shows side-by-side snapshots of the two apps for typical trials of the Gorilla tape rolling down the ramp. Regardless of which app is selected, the diameter of the rolling object must be measured and used in the analysis.

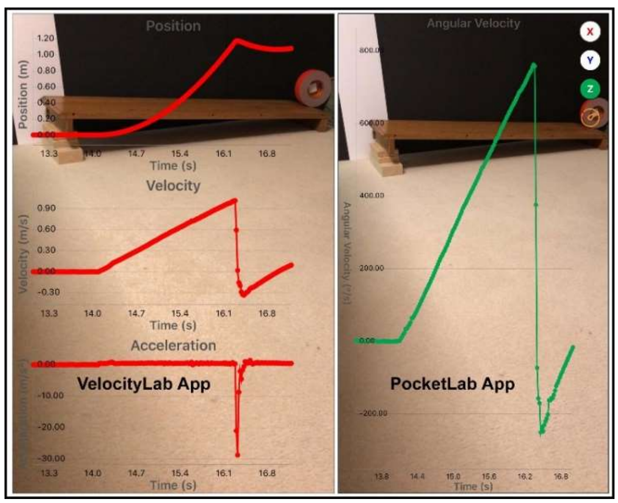

Figure 1

With the VelocityLab app, the diameter must be entered as part of the setup procedure when the app is started. But this app has the advantage that it then displays a graph of velocity of the center of mass *directly* as a function of time while the object rolls down the ramp. A possible minor disadvantage is that graphs of position and acceleration vs time are also provided, though they are not needed in the analysis. It should be noted that the VelocityLab app csv file contains columns for both translational velocity and angular velocity.

With the PocketLab app, the angular velocity is the item of interest. Students can obtain the angular velocity at the bottom of the ramp in ^{o}/s directly from the resultant graph, but then it is necessary to calculate the translational speed v of the center of mass by the equation $v = \omega R$, where ω is the angular velocity and R is the radius of the cylinder. Students also need to remember to convert from ^{o}/s to radians/s. ($180^{o} = \pi$ radians.)

The above considerations suggest that the VelocityLab app would likely provide for the easiest and quickest analysis of the experimental data. The author elected to use VelocityLab.

Specify any important assumptions you are making for data collected with your lab setup.

One should always be conscious of the possibility that a measuring device could change what is being measured. Clearly, attaching PocketLab to any of our three objects changes the geometry of the object. This could result in altering the translational velocity or angular velocity measurements for which PocketLab is responsible. With regard to the objects used by the author, the mass of PocketLab (17 gm) was only 2.3% of the mass of the Gorilla tape, 3.7% of the mass of the can of jellied cranberry sauce, and 11.4% of the mass of the cardboard tube. With these facts in mind, the author has assumed that the presence of PocketLab on the rolling objects would result in negligible alteration of translational and angular velocity.

Using conservation of mechanical energy, derive equations for the speed of the center of mass of the solid can and the hollow tube when they reach the bottom of the ramp. You will need to do a Web search for the moment of inertia of a solid cylinder and hollow cylinder about their central axis. According to the theory, is the speed dependent upon the radius and mass of these two objects?

Although we could derive these equations using dynamics, it is much simpler and direct to use conservation of mechanical energy. See Figure 2 for a diagram specifying the parameters involved.

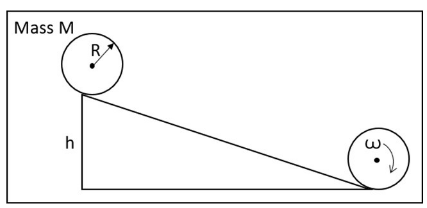

Figure 2

Because of static friction between the ramp and the objects, the objects roll instead of sliding down the ramp. The cylinders are initially at rest. While rolling down the ramp, *gravitational potential energy* is converted to *kinetic energy of rotation* as well as *kinetic energy of translation* of the center of mass:

$$Mgh = \tfrac{1}{2}I\omega^2 + \tfrac{1}{2}Mv^2. \quad \textit{(Equation 1)}$$

I is the moment of inertia of the object, ω is the angular velocity at the bottom of the ramp, and v is the speed of the center of mass of the object at the bottom of the ramp. The linear speed v and the angular velocity ω are related by the equation

$$\omega = v / R. \quad \textit{(Equation 2)}$$

Web research reveals the following moments of inertia for rotation about the central axis of each cylinder:

$$I = MR^2 \text{ (hollow tube)} \quad \text{and} \quad I = \tfrac{1}{2}MR^2 \text{ (solid can)}. \quad \textit{(Equation 3)}$$

Using Equations 1, 2, and 3 in conjunction with a little algebra, we readily find the following equations for the speed of the center of mass at the bottom of the ramp:

v = SQRT(gh) (hollow tube) and v = SQRT(4gh/3) (solid can). *(Equation 4)*

The solid can will have a somewhat higher translational speed than the hollow tube and would win in a ramp race between the two objects. We also see that according to theory, the translational speed at the bottom of the ramp is independent of the mass of the objects as well as their radii.

How do the theoretical speeds of the can and hollow tube compare to those you obtained from your experiment? What are some possible reasons for differences between your experimental and theoretical results?

Figure 3 shows a graph of velocity at the bottom of the ramp vs. time for 10 trials of the hollow cylinder rolling down the ramp. The data was obtained from the csv file produced by the VelocityLab app. The speed at the bottom of the ramp has been identified and shown for each trial. A similar graph, not shown here, was created for the solid can.

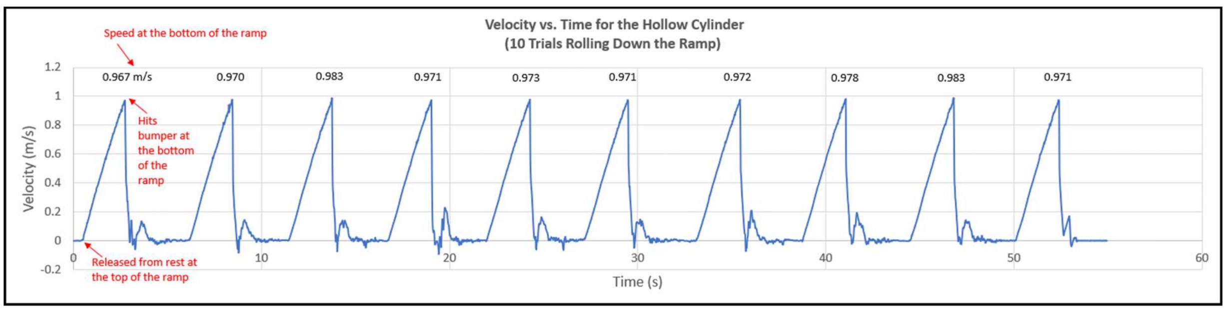

Figure 3

The table of Figure 4 summarizes the results for the solid can and hollow tube center of mass speeds at the bottom of the ramp, as obtained by the author. The height of the ramp used was 0.103 m.

	Trial 1	Trail 2	Trial 3	Trial 4	Trial 5	Trial 6	Trial 7	Trial 8	Trial 9	Trial 10	Average Experiment Speed (m/s)	Theoretical Speed Equation 4 (m/s)	%-Difference (expt - theory)
Can	1.149	1.156	1.154	1.158	1.158	1.153	1.154	1.143	1.150	1.158	1.153	1.16	-0.6%
Tube	0.967	0.970	0.983	0.971	0.973	0.971	0.972	0.978	0.983	0.971	0.974	1.00	-2.6%

Figure 4

Note that the results from one trial to the next are fairly consistent. This suggests that students could probably get by with as few as five trials for each object. Negative percent differences tell us that for both objects, the experimental speed was less than the theoretical speed. This suggests that there may be some dissipative forces such as air friction.

In a manner similar to that from challenge 2, obtain a formula for the theoretical speed of the center of mass of the Gorilla tape when it reaches the bottom of the ramp. Does this speed depend upon knowing the exact value of the inner and outer radius, or simply on knowing the ratio of these radii? Note that your equation for the Gorilla tape should reduce to the theoretical equation that you obtained in challenge 2 for the speed of the solid can if the inner radius is zero. Also note that your equation for the Gorilla tape should reduce to the theoretical equation you obtained in challenge 2 for the speed of the hollow cylinder if the inner radius is equal to the outer radius.

Web research reveals that the formula for the moment of inertial of an annular cylinder of mass M, inner radius R1, and outer radius R2 is given by:

$$I = \frac{1}{2}M(R_1^2 + R_2^2) \quad \text{(Equation 5)}$$

After a little algebra combining equations 1, 2, and 5, the following equation can be derived for the theoretical velocity of the center of mass of an annular cylinder at the bottom of the ramp:

$$v = \sqrt{\frac{4gh}{\left(\frac{R_1}{R_2}\right)^2 + 3}} \qquad \text{(Equation 6)}$$

Equation 6 clearly indicates that the speed is dependent on knowing only upon the ratio of inner and outer radius and not the exact values for each of the radii. Equation 6 reduces to equation 4 for the solid can if the inner radius is zero. Equation 6 also reduces to equation 4 for the hollow tube if the inner radius is equal to the outer radius. In effect, we see that the solid can and the hollow cylinder are but special cases of an annular cylinder as far moments of inertia about the central axis are concerned.

How does the theoretical speed of the Gorilla tape roll compare to those you obtained from your experiment? What are some possible reasons for differences between your experimental and theoretical results?

Figure 5 summarizes the results obtained by the author for all three objects. It is interesting that the Gorilla tape percent difference is so much greater than the can and the tube. A possible untested explanation is based upon the fact that the author noted the Gorilla tape roll is very slightly “squishy” due to all of the sticky stuff on the tape. This may result in the loss of energy as it rolls down the ramp. If the author could find a metal annular cylinder, it is hypothesized that the percent difference would be much less.

Since all three of the percent differences are negative, the experimental speeds are all somewhat less than the theoretical speeds. Some energy is lost due to possible dissipative forces including air resistance.

	Average Experiment Speed (m/s)	Theoretical Speed Equation 4 (m/s)	%-Difference (expt - theory)
Can	1.153	1.16	-0.6%
Tube	0.974	1.00	-2.6%
Gorilla Tape	1.023	1.115	-8.2%

Figure 5

Here are YouTube links to combined data/video for the Gorilla tape rolling down the ramp:

Velocity Lab App: https://youtu.be/4S8okWdJTUg

PocketLab App: https://youtu.be/QH6mlQuECOk